

Bachelor of Arts (B.A.) Part—I First Semester Examination**MATHEMATICS****(M₂ : Calculus)****Optional Paper—II**

Time : Three Hours]

[Maximum Marks : 60

N.B. :— (1) Solve all the **five** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.**UNIT—I**1. (A) By using $(\epsilon - \delta)$ definition of limit, show that :

$$\lim_{x \rightarrow 2} x^2 = 4.$$

6

(B) Discuss the continuity of the function :

$$f(x) = \begin{cases} x & , \quad x < 1 \\ 2 - x & , \quad 1 \leq x \leq 2 \\ -2 + 3x - x^2 & , \quad x > 2 \end{cases}$$

at $x = 1$ and $x = 2$.

6

OR(C) If f is finitely derivable at C , then prove that f is also continuous at C . Give an example to show that its converse is not true. 6(D) If $y = e^{a \sin^{-1} x}$, then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

6

UNIT—II2. (A) Expand $\log(1 + x)$ by Maclaurin's Theorem. 6

(B) Find the radius of curvature at any point on the curve :

$$x = a(\cos t + t \sin t), y = a(\sin t - t \cos t).$$

6

OR

(C) Find the asymptotes of the cubic curve :

$$2x^3 - x^2y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0. \quad 6$$

(D) Determine :

$$\lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{1}{\log(x-1)} \right]. \quad 6$$

UNIT—III

3. (A) If $u = \left[\sqrt{(x^2 + y^2 + z^2)} \right]^{-1}$; $x^2 + y^2 + z^2 \neq 0$, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. 6

(B) Find $\frac{dz}{dt}$ when, $z = xy^2 + x^2y$, $x = at^2$; $y = 2at$. Verify the result by direct substitution. 6

OR

(C) If $z = f(x, y)$ be a homogeneous function of degree n , then prove that :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz. \quad 6$$

(D) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \theta$, then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta. \quad 6$$

UNIT—IV

4. (A) Evaluate $\int \frac{2x+5}{\sqrt{x^2+3x+1}} dx$. 6

(B) Evaluate $\int \frac{1}{(x^2+1)\sqrt{(x^2-1)}} dx$. 6

OR

(C) Prove that $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, then show that :

$$I_n + I_{n-2} = \frac{1}{n-1}. \quad 6$$

(D) Show that :

$$\int_0^{\pi} \log(1 + \cos x) dx = \pi \log\left(\frac{1}{2}\right).$$

6

UNIT—V

5. (A) Show that the function $f(x) = (1 + 3x)^{1/x}$ when $x \neq 0$, $f(0) = e^3$ is continuous for $x = 0$. $1\frac{1}{2}$
- (B) Show that $f(x) = x |x|$ is derivable at $x = 0$. $1\frac{1}{2}$
- (C) Expand $f(x) = 5x^2 - 7x + 2$ in powers of $(x - 1)$. $1\frac{1}{2}$
- (D) Determine $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)$ $1\frac{1}{2}$
- (E) If $z = e^{2x} \sin 3y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. $1\frac{1}{2}$
- (F) If $z = \frac{x^2 + y^2}{x - y}$, then show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$. $1\frac{1}{2}$
- (G) Show that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{2}$. $1\frac{1}{2}$
- (H) Evaluate $\int \frac{1}{\sqrt{2 + 3x - x^2}} dx$. $1\frac{1}{2}$